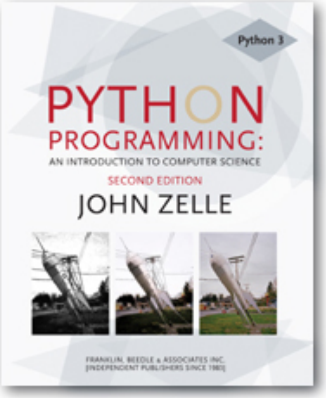
Python Programming: An Introduction to Computer Science



Chapter 3
Computing with Numbers

Objectives

- To understand the concept of data types.
- To be familiar with the basic numeric data types in Python.
- To understand the fundamental principles of how numbers are represented on a computer.

Objectives (cont.)

- To be able to use the Python math library.
- To understand the accumulator program pattern.
- To be able to read and write programs that process numerical data.

Numeric Data Types

- The information that is stored and manipulated bu computers programs is referred to as *data*.
- There are two different kinds of numbers!
  - (5, 4, 3, 6) are whole numbers – they don't have a fractional part
  - (.25, .10, .05, .01) are decimal fractions

Numeric Data Types

- Inside the computer, whole numbers and decimal fractions are represented quite differently!
- We say that decimal fractions and whole numbers are two different *data types*.

- The data type of an object determines what values it can have and what operations can be performed on it.

Numeric Data Types

- Whole numbers are represented using the *integer* (*int* for short) data type.
- These values can be positive or negative whole numbers.

Numeric Data Types

- Numbers that can have fractional parts are represented as *floating point* (or *float*) values.
- How can we tell which is which?
  - A numeric literal without a decimal point produces an int value
  - A literal that has a decimal point is represented by a float (even if the fractional part is 0)

Numeric Data Types

- Python has a special function to tell us the data type of any value.

```
>>> type(3)
<class 'int'>
>>> type(3.1)
<class 'float'>
>>> type(3.0)
<class 'float'>
>>> myInt = 32
>>> type(myInt)
<class 'int'>
>>>
```

Numeric Data Types

- Why do we need two number types?
  - Values that represent counts can't be fractional (you can't have 3 ½ quarters)
  - Most mathematical algorithms are very efficient with integers
  - The float type stores only an *approximation* to the real number being represented!
  - Since floats aren't exact, use an int whenever possible!

Numeric Data Types

- Operations on ints produce ints, operations on floats produce floats (except for /).

```
>>> 3.0+4.0
7.0
>>> 3+4
7
>>> 3.0*4.0
12.0
>>> 3*4
12
>>> 10.0/3.0
3.3333333333333335
>>> 10/3
3.3333333333333335
>>> 10 // 3
3
>>> 10.0 // 3.0
3.0
```

Numeric Data Types

- Integer division produces a whole number.
- That's why 10//3 = 3!
- Think of it as 'gozinta', where 10//3 = 3 since 3 gozinta (goes into) 10 3 times (with a remainder of 1)
- 10%3 = 1 is the remainder of the integer division of 10 by 3.
- a = (a/b)(b) + (a%b)

Using the Math Library

- Besides (+, -, *, /, //, **, %, abs), we have lots of other math functions available in a *math library*.
- A *library* is a module with some useful definitions/functions.

Using the Math Library

- Let's write a program to compute the roots of a quadratic equation!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The only part of this we don't know how to do is find a square root… but it's in the math library!

Using the Math Library

- To use a library, we need to make sure this line is in our program:
  import math
- Importing a library makes whatever functions are defined within it available to the program.

Using the Math Library

- To access the sqrt library routine, we need to access it as *math.sqrt(x)*.
- Using this dot notation tells Python to use the sqrt function found in the math library module.
- To calculate the root, you can do discRoot = math.sqrt(b*b – 4*a*c)

Using the Math Library

```
# quadratic.py
#    A program that computes the real roots of a quadratic equation.
#    Illustrates use of the math library.
#    Note: This program crashes if the equation has no real roots.

import math  # Makes the math library available.

def main():
    print("This program finds the real solutions to a quadratic")
    print()

    a, b, c = eval(input("Please enter the coefficients (a, b, c): "))

    discRoot = math.sqrt(b * b - 4 * a * c)
    root1 = (-b + discRoot) / (2 * a)
    root2 = (-b - discRoot) / (2 * a)

    print()
    print("The solutions are:", root1, root2 )

main()
```

Using the Math Library

```
This program finds the real solutions to a quadratic

Please enter the coefficients (a, b, c): 3, 4, -1

The solutions are: 0.215250437022 -1.54858377035
```

- What do you suppose this means?

```
This program finds the real solutions to a quadratic

Please enter the coefficients (a, b, c): 1, 2, 3

Traceback (most recent call last):
  File "<pyshell#26>", line 1, in -toplevel-
    main()
  File "C:\Documents and Settings\Terry\My Documents\Teaching\W04\CS
      120\Textbook\code\chapter3\quadratic.py", line 14, in main
    discRoot = math.sqrt(b * b - 4 * a * c)
ValueError: math domain error
>>>
```

Math Library

- If a = 1, b = 2, c = 3, then we are trying to take the square root of a negative number!
- Using the sqrt function is more efficient than using **. How could you use ** to calculate a square root?

Accumulating Results: Factorial

- Say you are waiting in a line with five other people. How many ways are there to arrange the six people?
- 720 -- 720 is the factorial of 6 (abbreviated 6!)
- Factorial is defined as:
  $n! = n(n-1)(n-2)\dots(1)$
- So, 6! = 6*5*4*3*2*1 = 720

Accumulating Results: Factorial

- How we could we write a program to do this?
- Input number to take factorial of, n
  Compute factorial of n, fact
  Output fact

Accumulating Results: Factorial

- How did we calculate 6!?
- 6*5 = 30
- Take that 30, and 30 * 4 = 120
- Take that 120, and 120 * 3 = 360
- Take that 360, and 360 * 2 = 720
- Take that 720, and 720 * 1 = 720

Accumulating Results: Factorial

- What's really going on?
- We're doing repeated multiplications, and we're keeping track of the running product.
- This algorithm is known as an *accumulator*, because we're building up or *accumulating* the answer in a variable, known as the *accumulator variable*.

Accumulating Results: Factorial

- The general form of an accumulator algorithm looks like this:

Initialize the accumulator variable

Loop until final result is reached
    update the value of accumulator variable

Accumulating Results: Factorial

- It looks like we'll need a loop!

```
fact = 1
for factor in [6, 5, 4, 3, 2, 1]:
    fact = fact * factor
```

- Let's trace through it to verify that this works!

Accumulating Results: Factorial

- Why did we need to initialize fact to 1? There are a couple reasons…
  - Each time through the loop, the previous value of fact is used to calculate the next value of fact. By doing the initialization, you know fact will have a value the first time through.
  - If you use fact without assigning it a value, what does Python do?

Accumulating Results: Factorial

- Since multiplication is associative and commutative, we can rewrite our program as:

```
fact = 1
for factor in [2, 3, 4, 5, 6]:
    fact = fact * factor
```

- Great! But what if we want to find the factorial of some other number??

Accumulating Results: Factorial

- What does *range(n)* return?
  0, 1, 2, 3, …, n-1
- range has another optional parameter!
  range(start, n) returns
  start, start + 1, …, n-1
- But wait! There's more!
  range(start, n, step)
  start, start+step, …, n-1
- list(<sequence>) to make a list

Accumulating Results: Factorial

- Let's try some examples!

```
>>> list(range(10))
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
>>> list(range(5,10))
[5, 6, 7, 8, 9]
>>> list(range(5,10,2))
[5, 7, 9]
```

Accumulating Results: Factorial

- Using this souped-up *range* statement, we can do the range for our loop a couple different ways.
  - We can count up from 2 to n: range(2, n+1)
    (Why did we have to use n+1?)
  - We can count down from n to 2: range(n, 1, -1)

Accumulating Results: Factorial

- Our completed factorial program:

```
# factorial.py
#    Program to compute the factorial of a number
#    Illustrates for loop with an accumulator

def main():
    n = eval(input("Please enter a whole number: "))
    fact = 1
    for factor in range(n,1,-1):
        fact = fact * factor
    print("The factorial of", n, "is", fact)

main()
```

The Limits of Int

- What is 100!?

```
>>> main()
Please enter a whole number: 100
The factorial of 100 is
    9332621544394415268169923885626670049071596826438162
    1468592963895217599993229915608941463976156518286253
    697920827223758251185210916864000000000000000000000
    00
```

- Wow! That's a pretty big number!

The Limits of Int

- Newer versions of Python can handle it, but…

```
Python 1.5.2 (#0, Apr 13 1999, 10:51:12) [MSC 32 bit (Intel)] on win32
Copyright 1991-1995 Stichting Mathematisch Centrum, Amsterdam
>>> import fact
>>> fact.main()
Please enter a whole number: 13
13
12
11
10
9
8
7
6
5
4
Traceback (innermost last):
  File "<pyshell#1>", line 1, in ?
    fact.main()
  File "C:\PROGRA~1\PYTHON~1.2\fact.py", line 5, in main
    fact=fact*factor
OverflowError: integer multiplication
```

The Limits of Int

- What's going on?
  - While there are an infinite number of integers, there is a finite range of ints that can be represented.
  - This range depends on the number of *bits* a particular CPU uses to represent an integer value. Typical PCs use 32 bits.

The Limits of Int

- Typical PCs use 32 bits
- That means there are 2^{32} possible values, centered at 0.
- This range then is -2^{31} to $2^{31}-1$. We need to subtract one from the top end to account for 0.
- But our 100! is much larger than this. How does it work?

Handling Large Numbers

- Does switching to *float* data types get us around the limitations of *int*s?
- If we initialize the accumulator to 1.0, we get

```
>>> main()
Please enter a whole number: 15
The factorial of 15 is 1.307674368e+012
```

- We no longer get an exact answer!

Handling Large Numbers: Long Int

- Very large and very small numbers are expressed in *scientific* or *exponential notation*.
- 1.307674368e+012 means 1.307674368 * 10^{12}
- Here the decimal needs to be moved right 12 decimal places to get the original number, but there are only 9 digits, so 3 digits of precision have been lost.

Handling Large Numbers

- Floats are approximations
- Floats allow us to represent a larger range of values, but with lower precision.
- Python has a solution, expanding ints!
- Python Ints are not a fixed size and expand to handle whatever value it holds.

Handling Large Numbers

- Newer versions of Python automatically convert your ints to expanded form when they grow so large as to overflow.
- We get indefinitely large values (e.g. 100!) at the cost of speed and memory

Type Conversions

- We know that combining an int with an int produces an int, and combining a float with a float produces a float.
- What happens when you mix an int and float in an expression?
  x = 5.0 + 2
- What do you think should happen?

Type Conversions

- For Python to evaluate this expression, it must either convert 5.0 to 5 and do an integer addition, or convert 2 to 2.0 and do a floating point addition.
- Converting a float to an int will lose information
- Ints can be converted to floats by adding ".0"

Type Conversion

- In *mixed-typed expressions* Python will convert ints to floats.
- Sometimes we want to control the type conversion. This is called *explicit typing*.

Type Conversions

```
>>> float(22//5)
4.0
>>> int(4.5)
4
>>> int(3.9)
3
>>> round(3.9)
4
>>> round(3)
3
```